

## 6.5

# Trapezoids

**Goal** Use properties of trapezoids.

### VOCABULARY

**Trapezoid** A trapezoid is a quadrilateral with exactly one pair of parallel sides.

**Bases of a trapezoid** The parallel sides of a trapezoid are the bases.

**Legs of a trapezoid** The nonparallel sides of a trapezoid are the legs.

**Base angles of a trapezoid** If trapezoid  $ABCD$  has bases  $\overline{AB}$  and  $\overline{CD}$ , then there are two pairs of base angles:  $\angle A$  and  $\angle B$ , and  $\angle C$  and  $\angle D$ .

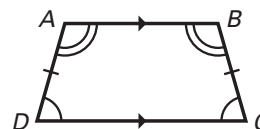
**Isosceles trapezoid** An isosceles trapezoid is a trapezoid with congruent legs.

**Midsegment of a trapezoid** The midsegment of a trapezoid is the segment that connects the midpoints of its legs.

### THEOREM 6.12

**Words** If a trapezoid is isosceles, then each pair of base angles is congruent.

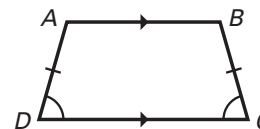
**Symbols** In the isosceles trapezoid  $ABCD$ ,  $\angle A \cong \angle B$  and  $\angle C \cong \angle D$ .



### THEOREM 6.13

**Words** If a trapezoid has a pair of congruent base angles, then it is isosceles.

**Symbols** In trapezoid  $ABCD$ , if  $\angle C \cong \angle D$  then  $ABCD$  is isosceles.



**Follow-Up** Tell whether the statement is true for *isosceles triangles*, *isosceles trapezoids*, *both*, or *neither*.

Legs are congruent. both

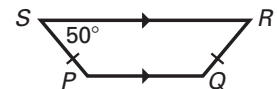
Base angles are congruent. both

Bases are parallel. isosceles trapezoids

Legs are parallel. neither

### Example 1 Find Angle Measures of Trapezoids

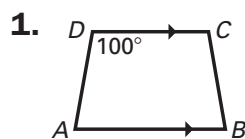
$PQRS$  is an isosceles trapezoid.  
Find the missing angle measures.



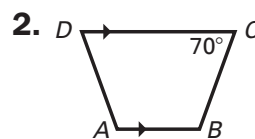
#### Solution

1. Use Theorem 6.12.  $PQRS$  is an isosceles trapezoid.  $\angle R$  and  $\angle S$  are a pair of base angles. So,  $m\angle R = m\angle S = 50^\circ$ .
2. Use the Same-Side Interior Angles Theorem. In the trapezoid,  $\overline{SR} \parallel \overline{PQ}$  cut by transversal  $\overline{SP}$ . Since  $\angle S$  and  $\angle P$  are same-side interior angles, they are supplementary. So,  $m\angle P = 180^\circ - 50^\circ = 30^\circ$ .
3. Use Theorem 6.12.  $PQRS$  is an isosceles trapezoid.  $\angle P$  and  $\angle Q$  are a pair of base angles. So,  $m\angle Q = m\angle P = 30^\circ$ .

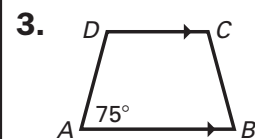
✓ **Checkpoint**  $ABCD$  is an isosceles trapezoid. Find the missing angle measures.



$$\begin{aligned} m\angle A &= 80^\circ; \\ m\angle B &= 80^\circ; \\ m\angle C &= 100^\circ \end{aligned}$$



$$\begin{aligned} m\angle A &= 110^\circ; \\ m\angle B &= 110^\circ; \\ m\angle D &= 70^\circ \end{aligned}$$

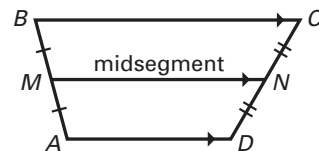


$$\begin{aligned} m\angle B &= 75^\circ; \\ m\angle C &= 105^\circ; \\ m\angle D &= 105^\circ \end{aligned}$$

## MIDSEGMENT OF A TRAPEZIOD

The length of the midsegment of a trapezoid is one half the sum of the lengths of the bases.

$$MN = \frac{1}{2}(AD + BC)$$



### Example 2 Midsegment of a Trapezoid

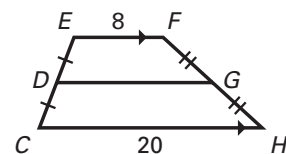
Find the length of the midsegment  $\overline{DG}$  of trapezoid  $CEFH$ .

#### Solution

Use the formula for the midsegment of a trapezoid.

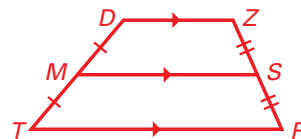
$$DG = \frac{1}{2}(EF + CH) = \frac{1}{2}(\underline{8} + \underline{20}) = \frac{1}{2}(\underline{28}) = \underline{14}$$

Answer The length of the midsegment  $\overline{DG}$  is 14.



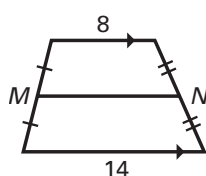
### Follow-Up

In the space at the right, draw any trapezoid  $TPZD$  with midsegment  $\overline{MS}$  so that the equation  $MS = \frac{1}{2}(TP + ZD)$  is true. Mark all parallel sides and congruent segments.



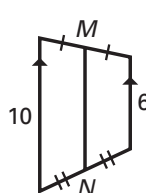
### ✓ Checkpoint Find the length of the midsegment $\overline{MN}$ .

4.



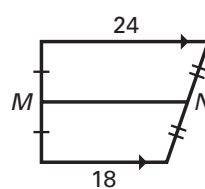
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